Approximated Peano Curve. The space-filling process has not been completed.

Figure 1A (Prior Art)
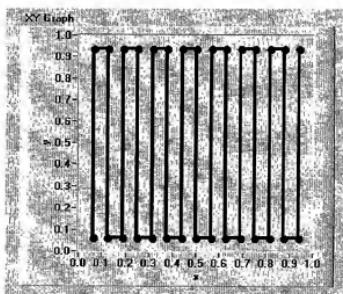
Boustrophedon Path

Figure 1B (Prior Art)
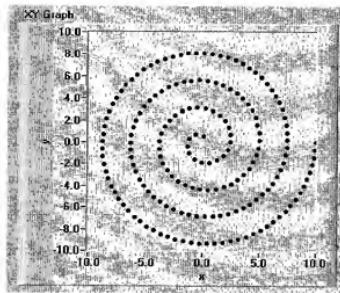
Archimedes Spiral defined by equally distributed points

Figure 1C (Prior Art)
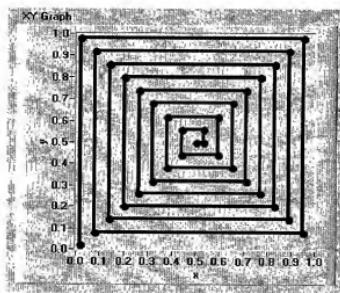
Spiral-like line-based scanning

Figure 1D (Prior Art)
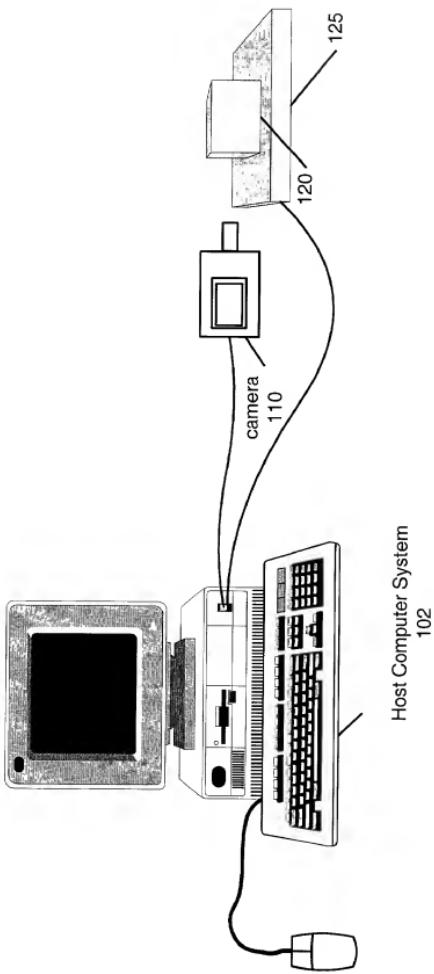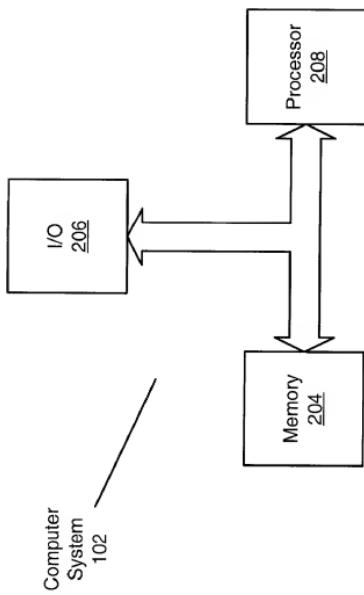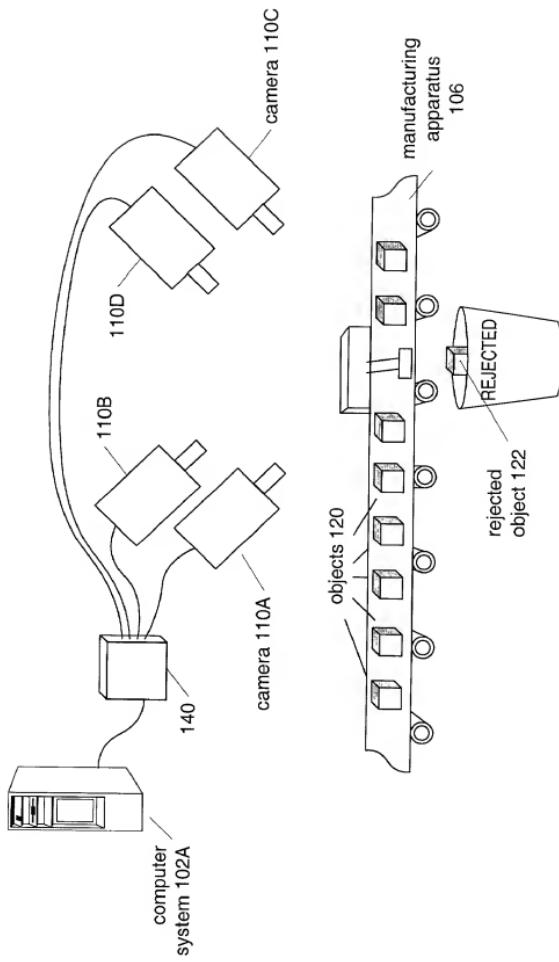
camera
110

120

125

Host Computer System
102

Figure 2A

I/O
206

Processor
208

Memory
204

Computer
System
102

Figure 2B

camera 110C

110D

110B

camera 110A

140

computer
system 102A

manufacturing
apparatus
106

objects 120

rejected
object 122

REJECTED

Figure 3A

camera
110F

robotic
arm
114B

camera
110E

120

robotic
arm
114A

computer
system 102B

Figure 3B

Computer System
102C

Phased Array
306

Array Elements
304

Figure 3C

Optical Fiber
320A

Laser Beam
312

Optical Fiber
320B

Motion Stage
330

Laser
310

Computer System
102D
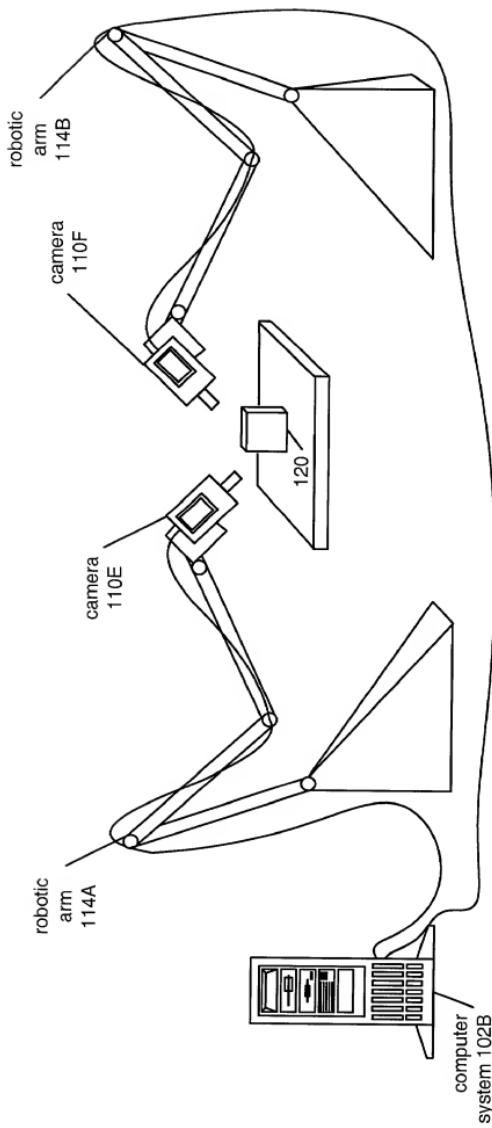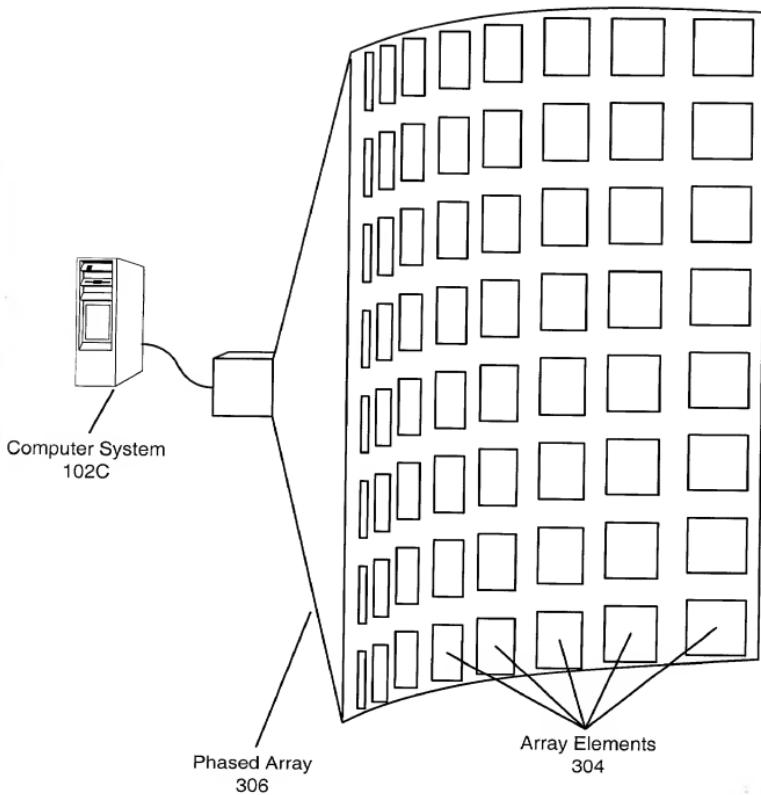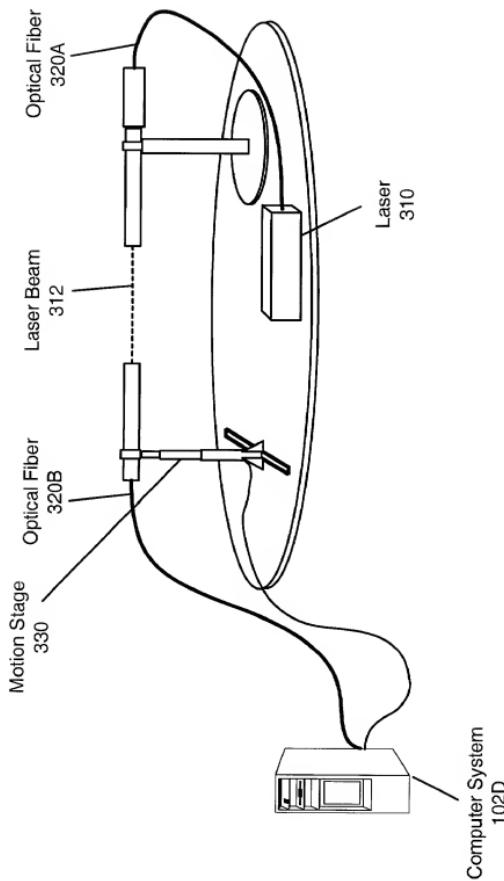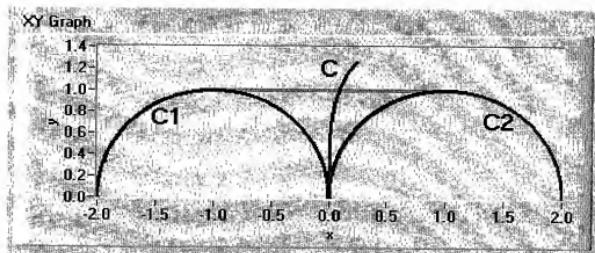
Figure 3D

The situation of Lemma 1

Figure 4A
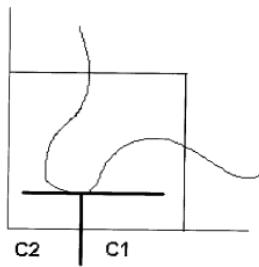
Case (A)
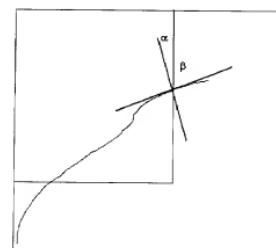
Figure 4B

Case (B)
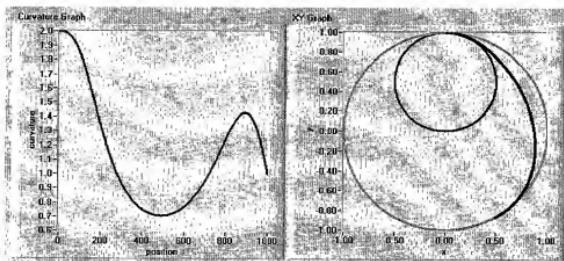
Figure 4C

Smooth transition between two circles of different radii.

Figure 4D
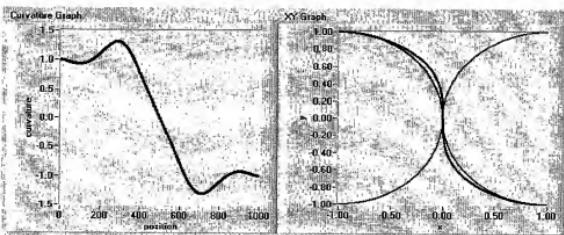
Smooth transition between two circles of same radius.

Figure 4E
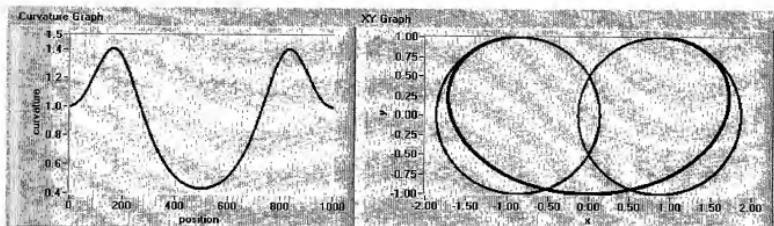
Transition between two unit circles of radius 1. The distance
between the circles is sqrt(3)
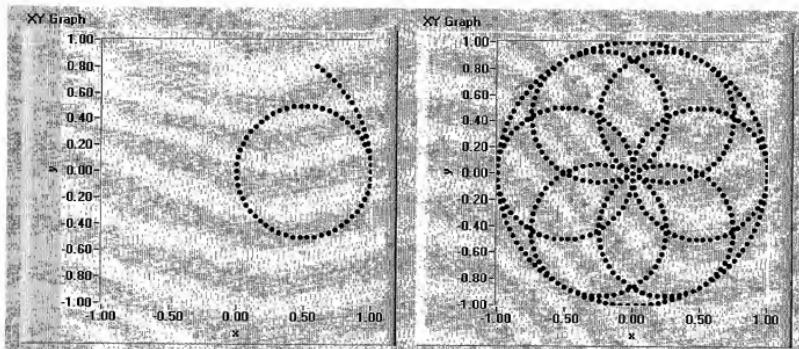
Figure 4F

Beginning (left) and completion (right) of a scanning scheme where the curvature is below a certain value
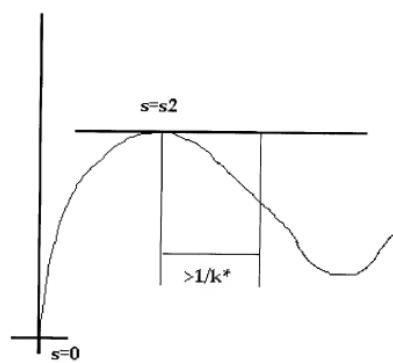
Figure 5A

Construction of s2 and the subsequent part of the curve

Figure 5B
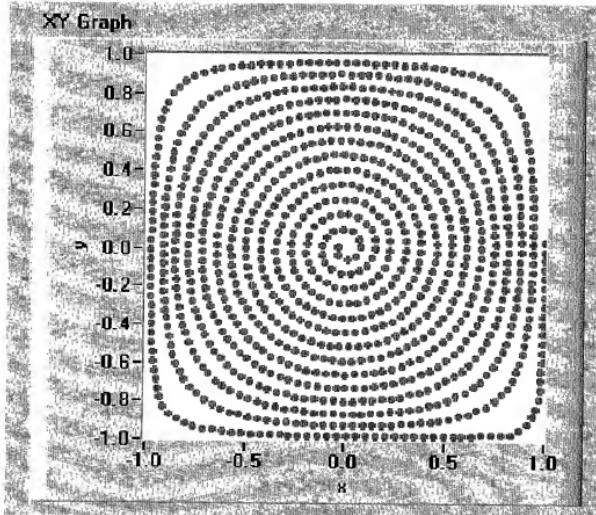
Conformal Spiral.

Figure 6

```
                        ┌─────────┐
                        │  Start  │
                        └────┬────┘
                             │
                             ▼
        ┌────────────────────────────────────────┐
        │        Place Object In Scan Region      │
        │                   702                    │
        └────────────────────┬───────────────────┘
                             │
                             ▼
        ┌────────────────────────────────────────┐
        │  Determine Characteristic Geometry of   │
        │             Scan Region                  │
        │                   704                    │
        └────────────────────┬───────────────────┘
                             │
                             ▼
        ┌────────────────────────────────────────┐
        │     Generate Conformal Scan Curve Based  │
        │         on Characteristic Geometry       │
        │                   706                    │
        └────────────────────┬───────────────────┘
                             │
                             ▼
        ┌────────────────────────────────────────┐
        │   Scan Region Using the Conformal Scan   │
        │                  Curve                   │
        │                   708                    │
        └────────────────────┬───────────────────┘
                             │
                             ▼
        ┌────────────────────────────────────────┐
        │ Determine One or More Characteristics of │
        │               the Object                 │
        │                   710                    │
        └────────────────────┬───────────────────┘
                             │
                             ▼
        ┌────────────────────────────────────────┐
        │ Generate Output Indicating the Determined│
        │           Object Characteristics         │
        │                   712                    │
        └────────────────────┬───────────────────┘
                             │
                             ▼
                        ┌─────────┐
                        │   End   │
                        └─────────┘
```
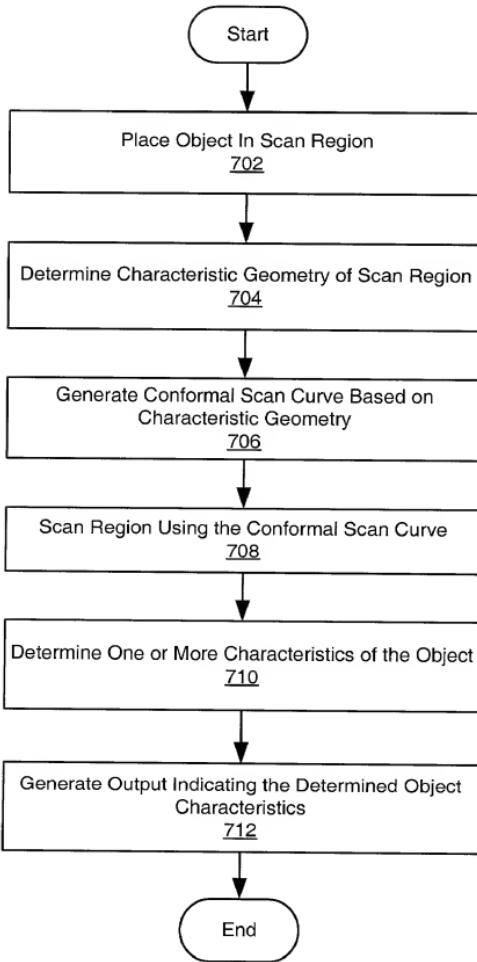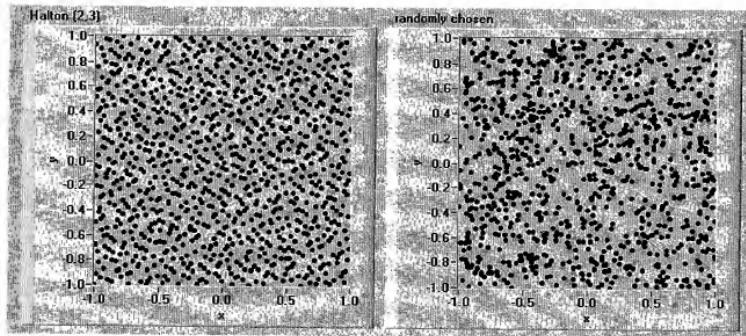
Figure 07

The first 1000 Halton points (left) and randomly chosen points (right)

Figure 8A
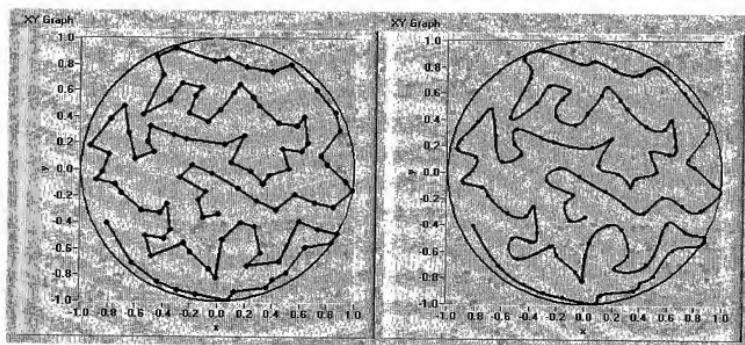
Original solution (left) and splined version (right).

Figure 8B

```
      ┌─────────┐
      │  Start  │
      └────┬────┘
           │
           ▼
┌──────────────────────────────────────┐
│ Calculate Low Discrepancy Sequence   │
│ for Scan Region                      │
│ 902                                  │
└──────────────┬───────────────────────┘
               │
               ▼
┌──────────────────────────────────────┐
│ Calculate a Motion Control Trajectory│
│ Based On the Low Discrepancy Sequence│
│ 904                                  │
└──────────────┬───────────────────────┘
               │
               ▼
┌──────────────────────────────────────┐
│ Scan Region Using Calculated Motion  │
│ Control Trajectory                   │
│ 906                                  │
└──────────────┬───────────────────────┘
               │
               ▼
┌──────────────────────────────────────┐
│ Determine Object Characteristics     │
│ 908                                  │
└──────────────┬───────────────────────┘
               │
               ▼
┌──────────────────────────────────────┐
│ Generate Output Indicating the       │
│ Determined Object Characteristics    │
│ 910                                  │
└──────────────┬───────────────────────┘
               │
               ▼
      ┌─────────┐
      │   End   │
      └─────────┘
```
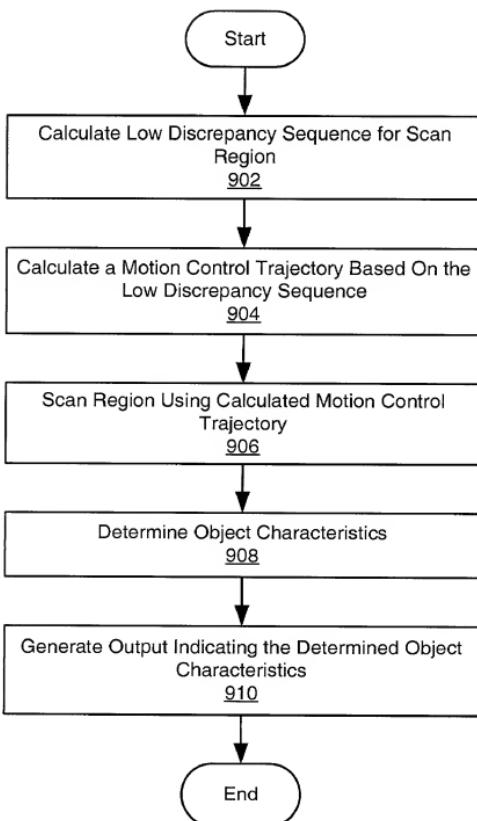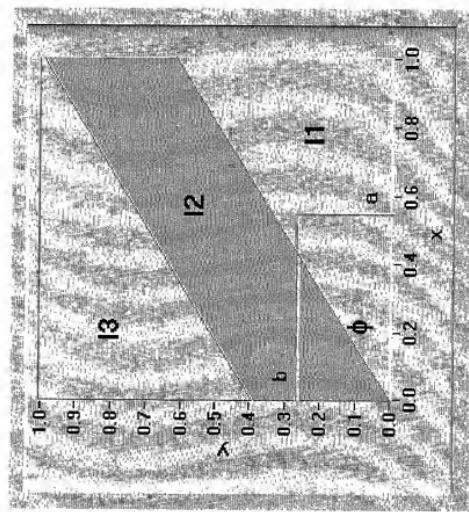
Figure 9

Definition of I_1, I_2, and I_3

Figure 10
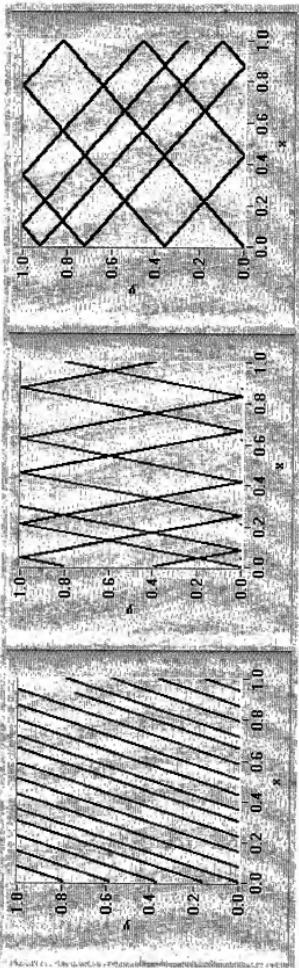
Figure 11A

Figure 11B

Figure 11C

```
          ┌─────────────────────────────────────────┐
          │                                         │◄─────────┐
          │   Generate Unbounded Low Discrepancy Point        │
          │                 1222                     │         │
          │                                         │         │
          └─────────────────────────────────────────┘         │
                            │                                  │
                            ▼                                  │
          ┌─────────────────────────────────────────┐         │
          │  Apply Boundary Conditions to the Unbounded Low    │
          │   Discrepancy Point to Generate a Bounded Low      │
          │           Discrepancy Point in a Region            │
          │                 1224                     │         │
          └─────────────────────────────────────────┘         │
                            │                                  │
                            ▼                                  │
                        ◇ Done? ◇         No                   │
                          1226      ──────────────────────────┘
                            │
                          Yes
                            │
                            ▼
          ┌─────────────────────────────────────────┐
          │  Store Generated Low Discrepancy Sequence of       │
          │ Points Representing a Low Discrepancy Curve in the │
          │                  Region                  │
          │                 1228                     │
          └─────────────────────────────────────────┘
                            │
                            ▼
          ┌─────────────────────────────────────────┐
          │  Output Generated Low Discrepancy Sequence of      │
          │  Points Representing the Low Discrepancy Curve     │
          │                 1230                     │
          └─────────────────────────────────────────┘
```

Figure 12A

```
┌─────────────────────────────────────────────┐
│ Select a Pair of Irrational Numbers (alpha1,alpha2) │
│   such that the Sequence ({(n*alpha1)mod1},    │
│ {(n*alpha2) mod1}) for all Natural Numbers n is a │
│            LDS in the Unit Square.             │
│                    1202                        │
└─────────────────────────────────────────────┘
                      │
                      ▼
┌─────────────────────────────────────────────┐
│  Select a Length, L, and a Step Rate, epsilon, of the │
│           LD Curve in the Unit Square           │
│                    1204                        │
└─────────────────────────────────────────────┘
                      │
                      ▼
┌─────────────────────────────────────────────┐
│  Initialize Current Length, I, to Zero, and Initialize │
│     Current Position (x ,y), (e.g., to (0,0)    │
│                    1206                        │
└─────────────────────────────────────────────┘
                      │
                      ▼
┌─────────────────────────────────────────────┐
│ Increment x and y and Apply Boundary Conditions at │
│  Borders of Unit Square (e.g., Toroidal, Reflectance, │
│  or Both), Generating a Low Discrepancy Sequence │
│              Point (xₙ,yₙ)                      │
│                    1208                        │
└─────────────────────────────────────────────┘
                      │
                      ▼
                  ◇ I < L? ◇      No
                  ◇  1210  ◇  ──────────►
                      │
                     Yes
                      ▼
┌─────────────────────────────────────────────┐
│ Output Generated Low Discrepancy Sequence of │
│  Points Representing the Low Discrepancy Curve │
│                    1212                        │
└─────────────────────────────────────────────┘
```

Figure 12B

Beginning of a Low Discrepancy Curve based on a specific
Halton Sequence in 2d

Figure 13A
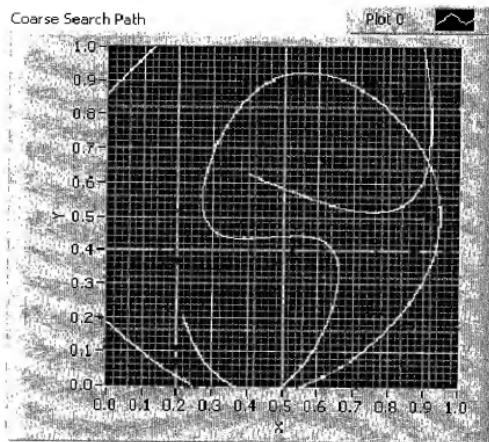
Splined Low Discrepancy Curve coarse search

Figure 13B
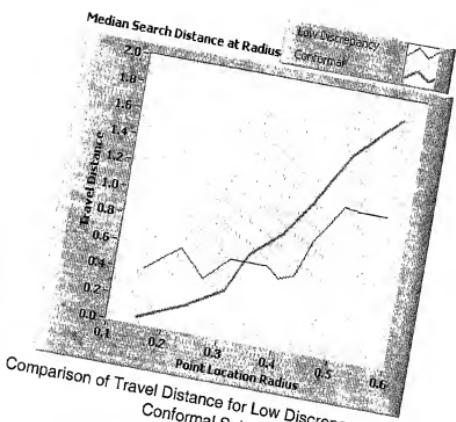
Comparison of Conformal Spiral and Low Discrepancy Searching

Figure 13C

Comparison of Travel Distance for Low Discrepancy Search and Conformal Spiral Search

Figure 13D

```
Start
  │
  ▼
```

Calculate Low Discrepancy Sequence for Scan
Region
1402

Calculate a Low Discrepancy Curve Based On the
Low Discrepancy Sequence
1404

Scan Region Using Calculated Low Discrepancy
Curve
1406

Determine Object Characteristics
1408

Generate Output Indicating the Determined Object
Characteristics
1410

```
  │
  ▼
```

End
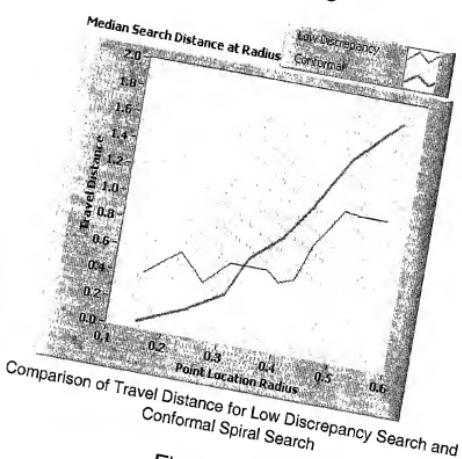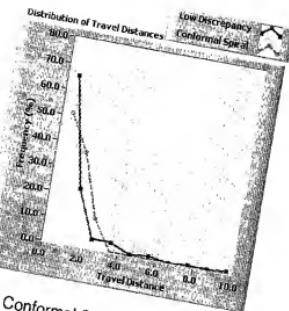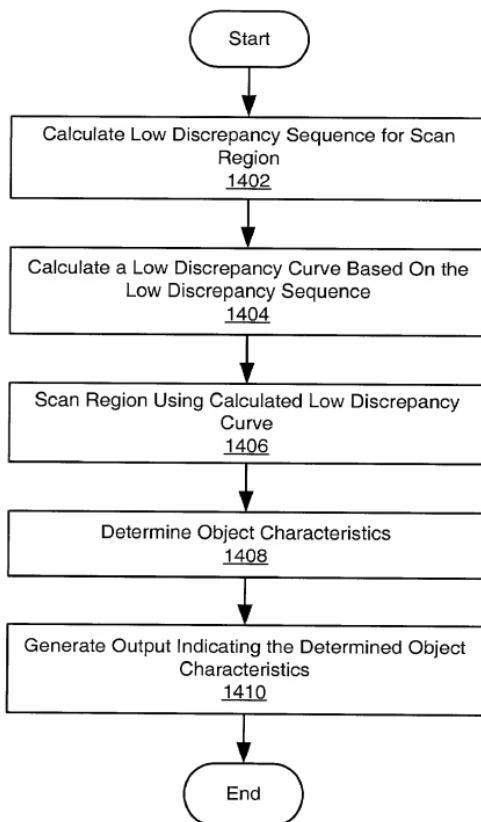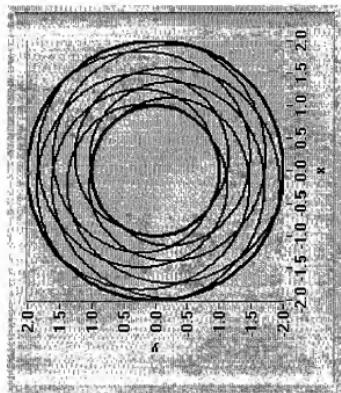
Figure 14

Low-discrepancy curve in a ring
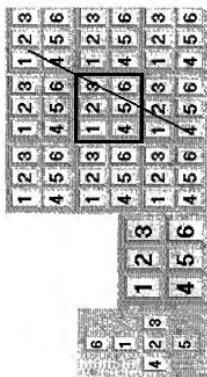
Figure 15B

Tiling of the plane and relation to the surface of the unit cube

Figure 15A

Low-discrepancy curve filling the surface of a torus
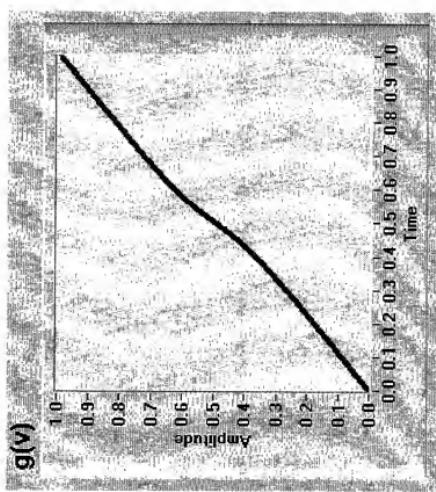
Figure 15D

Low Discrepancy Preserving Mapping Function
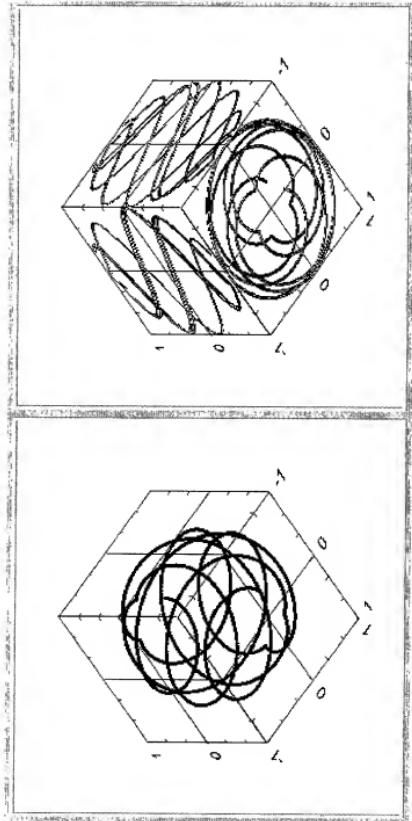
Figure 15C

Low-discrepancy curve on a sphere
(left) and projections (right)

Figure 16

```
┌─────────────────────────────────────────┐
│                                         │
│  Select a Parameterization of a Surface S Where the │
│      Parameter Space is the Unit Square  │
│                   1702                  │
│                                         │
└─────────────────────────────────────────┘
                    │
                    ▼
┌─────────────────────────────────────────┐
│                                         │
│   Select a LD Curve in the Parameter Space, i.e., │
│       Select a Suitable (alpha1, alpha2) │
│                   1704                  │
│                                         │
└─────────────────────────────────────────┘
                    │
                    ▼
┌─────────────────────────────────────────┐
│  Determine a Re-Parameterization of S Such That │
│ the Ratio of Line and Area Elements of S, Based on │
│         a Riemannian Metric, is Constant │
│                   1706                  │
└─────────────────────────────────────────┘
                    │
                    ▼
┌─────────────────────────────────────────┐
│                                         │
│   Map the LD Curve in the Unit Square onto the │
│     Surface S  Using the Re-Parameterization │
│                   1708                  │
│                                         │
└─────────────────────────────────────────┘
                    │
                    ▼
┌─────────────────────────────────────────┐
│                                         │
│         Output the Mapped LD Curve on S │
│                   1710                  │
│                                         │
└─────────────────────────────────────────┘
```

Figure 17

Surfaces can be scanned efficiently when the term low discrepancy sequence/
curve can be generalized, e.g. based on metrics on the surface.

Figure 18
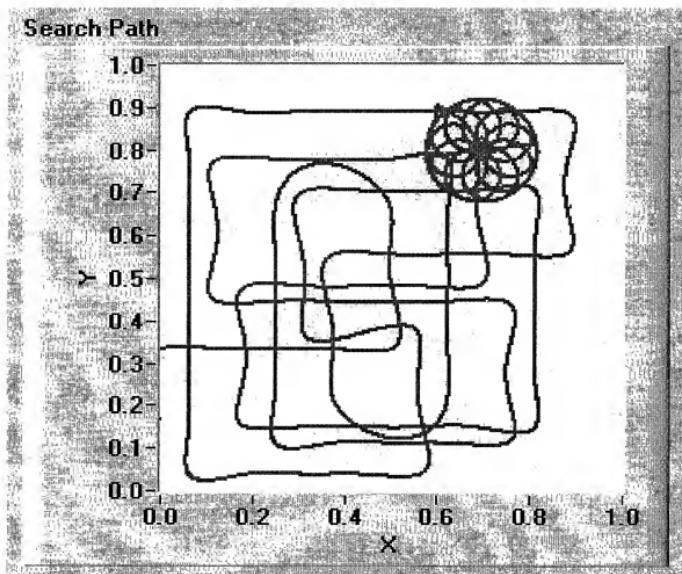
Splined Low Discrepancy Curve coarse search with refined final approach

Figure 19

Intensity Field Distribution in Search Area
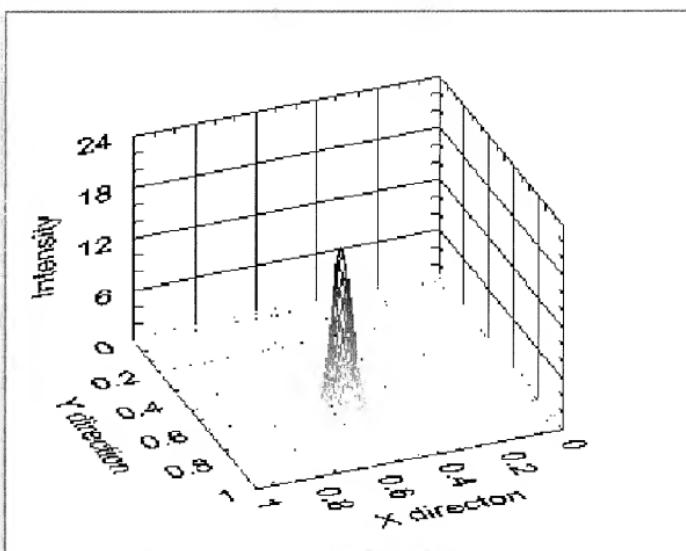
Beam intensity distribution in search area

Figure 20
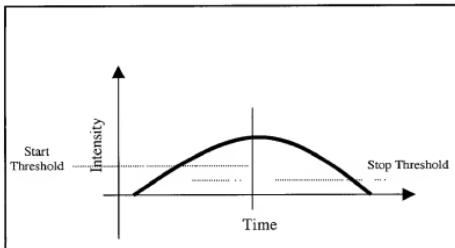
Location of the Peak
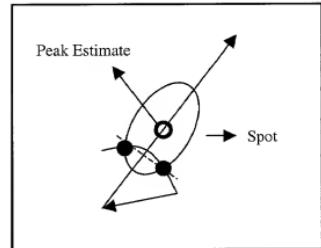
Figure 21A

Initial Final Approach Move

Figure 21B
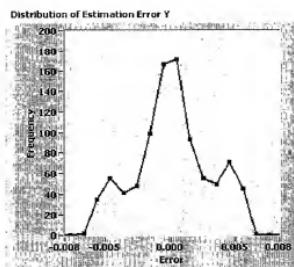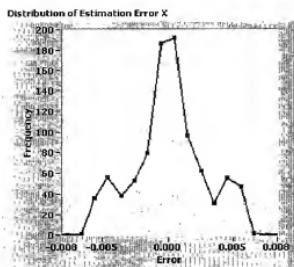
Error distribution of the estimated peak X coordinate error (left) and Y coordinate error (right)

Figure 21C

```
       ( Start )
           │
           ▼
┌──────────────────────────────────────┐
│ Determine Characteristics of Region  │
│        of Interest in the Region      │
│                 2202                   │
└──────────────────────────────────────┘
           │
           ▼
┌──────────────────────────────────────┐
│ Calculate a Continuous Scan Trajectory│
│ Based On the Determined Characteristics│
│        of the Region of Interest       │
│                 2204                   │
└──────────────────────────────────────┘
           │
           ▼
┌──────────────────────────────────────┐
│ Measure Region Along Continuous Scan  │
│  Trajectory to Generate a Sample Data │
│                 Set                    │
│                 2206                   │
└──────────────────────────────────────┘
           │
           ▼
┌──────────────────────────────────────┐
│ Fit the Sample Data Set With a        │
│ Parameterized Surface Using A Model   │
│           of the Region               │
│                 2208                   │
└──────────────────────────────────────┘
           │
           ▼
┌──────────────────────────────────────┐
│ Calculate Location of Point of Interest│
│     Based on Parameterized Surface     │
│                 2210                   │
└──────────────────────────────────────┘
           │
           ▼
┌──────────────────────────────────────┐
│ Measure the Region at the Point of    │
│  Interest to Confirm the Calculated   │
│              Location                 │
│                 2212                   │
└──────────────────────────────────────┘
           │
           ▼
┌──────────────────────────────────────┐
│ Generate Output Indicating the        │
│   Determined Object Characteristics   │
│                 2214                   │
└──────────────────────────────────────┘
           │
           ▼
        ( End )
```

Figure 22

```
                          ┌─────────┐
                          │  Start  │
                          └────┬────┘
                               │
                               ▼
        ┌──────────────────────────────────────────────┐
        │  Scan Region to Determine An Entry Point and An │
        │     Exit Point of the Region of Interest        │
        │                    2302                         │
        └──────────────────────┬───────────────────────┘
                               │
                               ▼
        ┌──────────────────────────────────────────────┐
        │ Determine First Local Point of Interest By Scanning │
        │ Along First Scan Line Between Entry and Exit Points │
        │                    2304                         │
        └──────────────────────┬───────────────────────┘
                               │
                               ▼
        ┌──────────────────────────────────────────────┐
        │ Calculate Second Scan Line Passing Through First │
        │ Local Point of Interest and Orthogonal to First Scan │
        │                    Line                         │
        │                    2306                         │
        └──────────────────────┬───────────────────────┘
                               │
                               ▼
        ┌──────────────────────────────────────────────┐
        │  Scan Along Second Scan Line to Generate Second │
        │        Scan Line Measurement Data               │
        │                    2308                         │
        └──────────────────────┬───────────────────────┘
                               │
                               ▼
        ┌──────────────────────────────────────────────┐
        │  Determine Second Local Point of Interest Along │
        │  Second Scan Line Based on Second Scan Line      │
        │              Measurement Data                   │
        │                    2310                         │
        └──────────────────────┬───────────────────────┘
                               │
                               ▼
        ┌──────────────────────────────────────────────┐
        │ Determine Approximate Center of the Region of   │
        │ Interest Based on First and Second Points of Interest │
        │                    2312                         │
        └──────────────────────┬───────────────────────┘
                               │
                               ▼
                          ┌─────────┐
                          │   End   │
                          └─────────┘
```

Figure 23